RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR [2015-18]
B.A./B.Sc. FIRST SEMESTER (July – December) 2015
Mid-Semester Examination, September 2015

Date : 14/09/2015 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate answer book for each group]

Group – A

Answer **any three** questions : **[3 × 4]**

1. Let F be a finite subset of $\mathbb{N}$. Does there exist a bijective map $f:\mathbb{N}\to\mathbb{N}-F$? Justify your answer. [4]

2. a) Let $\mathcal{A}=\{A\subseteq\mathbb{N}:\mathbb{N}-A \text{ is finite}\}$. Find $\cap\{A:A\in\mathcal{A}\}$.
   b) Let $A_n=\{x\in\mathbb{R}:x>n\}$ $(n\in\mathbb{N})$. Find $\bigcap\limits_{n=1}^{\infty}A_n$. [2 + 2]

3. Prove that the number of reflexive relations on a set containing n elements is 2^{n^2-n}. [4]

4. Define an equivalence relation on $X=\{1,2,3,4\}$ having three equivalence classes. [4]

5. Define a binary relation ρ on $\mathbb{R}$ as follows : '$a\rho b$ iff $a-b\in\mathbb{Q},(a,b\in\mathbb{R})$' Show that ρ is an equivalence relation. Find the equivalence class containing 0. [4]

Answer **Q.No. 6** and **any two** : [5 + 2 × 4]

6. a) Define a closed set. [1]
   b) Give example of a set which has exactly five limit points. [1]
   c) Give example of sets S of real numbers such that S is not an interval and (i) Sup (S) belongs to S and (ii) Sup (S) does not belongs to S. [2]
   d) Show that countable union of closed sets may not be closed. [1]

7. a) Prove that a set which is the complement of an open set is closed. [2]
   b) Is every closed set, complement of some open set? [2]

8. a) Prove that any superset of a dense set is dense again. [3]
   b) Give example of uncountably many dense subsets of $\mathbb{R}$, the set of all reals. [1]

9. a) Prove that the set of naturals $\mathbb{N}$ is equipotent with $\mathbb{N}^2$. [2]
   b) Find disjoint subsets $A_i\,(i=1,2,3,4,5,...)$ of set of naturals $\mathbb{N}$, such that their union is $\mathbb{N}$. [2]

Group – B

Answer **any two :** **[2 × 6]**

10. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting straight lines, show that the area formed by the bisectors of the angles between them and the x-axis is $\dfrac{1}{2}\dfrac{g^2-ca}{h^2-ab}\dfrac{\sqrt{(a-b)^2+4h^2}}{|h|}$. [6]

11. Show that the equation of the circle which passes through the focus of the parabola $\frac{2a}{r} = 1 + \cos\theta$ and touches it at the point $\theta = \alpha$ is given by $r^2 \cos^3 \frac{\alpha}{2} = a \cos\left(\theta - \frac{3\alpha}{2}\right)$. [6]

12. Reduce the equation $6x^2 + 24xy - y^2 - 60x - 20y + 80 = 0$ to its canonical form and state the nature of the conic represented by it. Also find the equations of its axes. [6]

13. Answer **any two** questions : [2×4]

a) Find the differential equation of a system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ where λ is the arbitrary parameter. [4]

b) Solve : $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$. [4]

c) Solve $\frac{dy}{dx} + 2y\tan x = \sin x$ when $x = \frac{\pi}{3}$, y = 0; show also that the maximum value of y is $\frac{1}{8}$. [4]

14. Answer **any one** question : [1×5]

a) Prove that the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ will be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [5]

b) Show that the substitution $u = x^2 + y^2$, $v = x^2 - y^2$, transforms the equation $(px - y)(x - py) = 2p$, where $p = \frac{dy}{dx}$, into Clairauts form. Also deduce the general and singular solutions of the original equation. [5]

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